Part 5: Dynamics

University Physics V1 **(Openstax): Chapters 5 and 6**
Physics for Engineers & Scientists **(Giancoli): Chapter4**

Forces

- Intuitively, a force is a push or a pull.
- Forces are vectors with both magnitude and direction.
- Contact forces are created when two objects are in physical contact.
- Non-contact forces are felt between objects that are not in contact.

Mass is a measure of the amount of matter in an object.

Newton's Laws of Motion

- **Newton's 1st Law of Motion**: **"The Law of Inertia"**

  Every object continues in its state of rest, or of uniform speed in a straight line, as long as no net force acts on it.

  - The tendency for an object to move at a constant speed in a straight line is called inertia.
  - Mass is a qualitative measure of the inertia of an object.
  - Velocity is a vector. A change in the speed or direction of motion is a change in inertia.
  - If something is speeding up, slowing down or changing direction Newton's First Law says there is an external FORCE acting on it.
  - An inertial reference frame is a coordinate system that obeys Newton's First Law. Any reference frame moving with constant velocity (or stationary) with respect to an inertia reference frame is also an inertia reference frame

- **Newton's 2nd Law of Motion**: "**The Law of Force and Acceleration.**"

  The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportionally to the object's mass. The direction of the acceleration is in the direction of the net force acting on the object.

  - The standard SI unit of mass is the kilogram (kg)
  - The standard SI unit of force is the Newton (N): $1\ N = 1\ kg \cdot m/s^2$.
  - This is a vector equation.
    1) The acceleration points in the same direction as the sum of the forces.
    2) The magnitude of the net force equals the mass multiplied by the magnitude of the acceleration.
  - There are two cases
    1) In static equilibrium the acceleration (and the sum of the forces) is zero.
    2) In non-equilibrium the acceleration is not zero (and neither is the sum of the forces)

  Newton's 2nd law acts as a link between force problems and kinematic problems.

- **Newton's 3rd Law of Motion**: Whenever one object exerts a force on a second object, the second exerts an equal force in the opposite direction.

  "For every action, there is an equal and opposite reaction."

The force that a nail experiences when hit with a hammer (driving it into the wood) is equal in magnitude and opposite in direction to the force that the hammer experiences from being hit by the nail, slowing the hammer down.

<u>Example</u>: An F-14 has a mass of $3.1\text{x}10^4$ kg and takes off under the influence of a constant net force of $3.7\ \text{x}10^4$ N. What is the net force that acts upon the 78 kg pilot?

The forces and accelerations in this problem are collinear. We may treat it 1-dimensionally.

$$\sum \vec{F} = m\vec{a} \qquad F_{F-14} = m_{F-14} \cdot a \qquad a = \frac{F_{F-14}}{m_{F-14}} = \frac{3.7\times10^4\ \text{N}}{3.1\times10^4\ \text{kg}} = 1.19355\ \text{m/s}^2$$

$$F_{Pilot} = m_{Pilot} \cdot a = (78\ \text{kg})\left(1.19355\frac{\text{m}}{\text{s}^2}\right) = 93.0969\ \text{N} \Rightarrow 93\ \text{N}$$

<u>Example</u>: When a 58g tennis ball is served, it accelerates from rest to a speed of 45m/s. The impact of the racket gives the ball a constant acceleration over a distance of 44cm. What is the magnitude of the net force acting on the ball?

The forces and accelerations in this problem are collinear. We may treat it 1-dimensionally.

The acceleration is needed to solve this problem (F=ma). We can find the acceleration from kinematics.

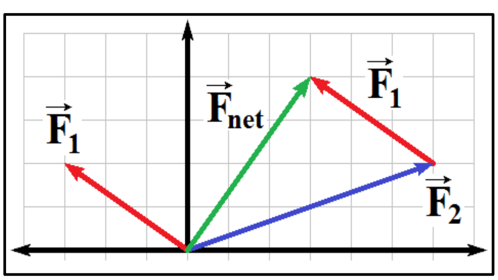
$$x_0 = 0 \qquad x = 44\ \text{cm} = 0.44\ \text{m} \qquad v_0 = 0 \qquad v = 45\ \text{m/s} \qquad a = ???$$

$$v^2 = v_0^2 + 2a(x - x_0) = 2ax \qquad a = \frac{v^2}{2x} = \frac{\left(45\frac{\text{m}}{\text{s}}\right)^2}{2(0.44\ \text{m})} = 2301.14\frac{\text{m}}{\text{s}^2}$$

$$F = ma = (0.058\ \text{kg})\left(2301.14\frac{\text{m}}{\text{s}^2}\right) = 133.466\ \text{N} \Rightarrow 130\ \text{N}$$

<u>Example</u>: Two forces, $\vec{F}_1 = (-3\ \text{N})\hat{\imath} + (2\ \text{N})\hat{\jmath}$ and $\vec{F}_2 = (6\ \text{N})\hat{\imath} + (2\ \text{N})\hat{\jmath}$ are acting on an object with a mass of 1 kg. What is the magnitude of that object's acceleration?

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = \underbrace{(-3\ \text{N})\hat{\imath} + (2\ \text{N})\hat{\jmath}}_{\vec{F}_1} + \underbrace{(6\ \text{N})\hat{\imath} + (2\ \text{N})\hat{\jmath}}_{\vec{F}_2} = (3\ \text{N})\hat{\imath} + (4\ \text{N})\hat{\jmath} = (5\ \text{N})\angle 53.13°$$



$$|\vec{F}_{net}| = \sqrt{F_{net-x}^2 + F_{net-y}^2} = \sqrt{(3\ \text{N})^2 + (4\ \text{N})^2} = 5\ \text{N}$$

$$\theta = \text{Tan}^{-1}\left(\frac{F_{net-y}}{F_{net-x}}\right) = \text{Tan}^{-1}\left(\frac{4\ \text{N}}{3\ \text{N}}\right) = 53.13°$$

$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{(5\ \text{N})\angle 53.13°}{1\ \text{kg}} = \left(5\frac{\text{m}}{\text{s}^2}\right)\angle 53.13°$$

Example: A marksman fires a rifle. The 9.50 gram bullet accelerates from rest to 851 m/s in 1.31 ms at which point it leaves the barrel of the rifle. What is the average recoil force on the gun from the bullet?

Determine the force on the bullet from its acceleration.
The magnitude of the force on the rifle is the same.

$x_0 = 0$ $\qquad x =$ $\qquad v_0 = 0$ $\qquad v = 851\text{ m/s}$ $\qquad a = ???$ $\qquad t = 0.00131\text{ s}$

$$v = v_0 + at = at \qquad a = v/t = (851\text{ m/s})/(0.00131\text{ s}) = 649{,}618\text{ m/s}^2$$

$$F = ma = (0.00950\text{ kg})(649{,}618\text{ m/s}^2) = 6171.37\text{ N} \Rightarrow 6.17\text{ kN}$$

Weight

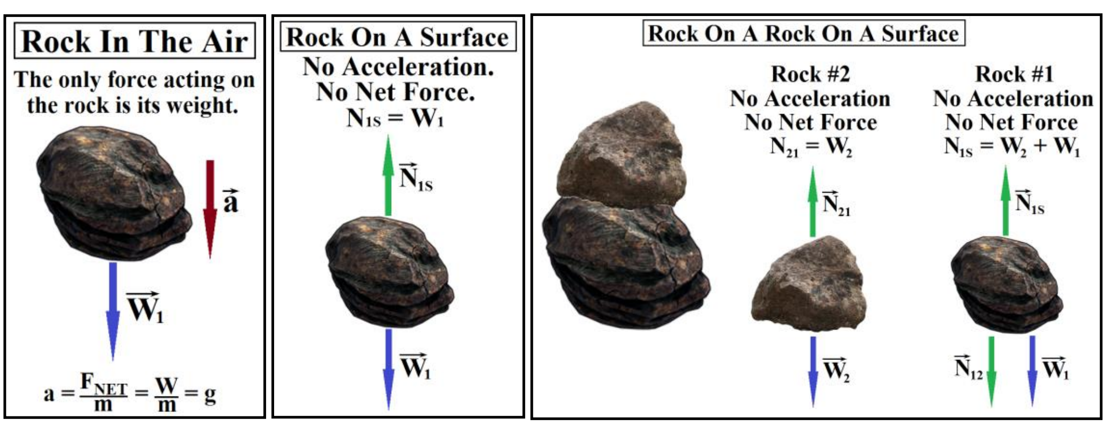
- Objects accelerate downward due to the gravitational force from the Earth.
- This force from gravity is called weight (W). $\vec{W} = m\vec{g}$
- Be careful not to confuse weight (a force) with mass (not a force).
  - The mass of objects in space doesn't change, but they have no weight.
  - On the moon, the gravitational acceleration is $g = 1.622\text{ m/s}^2$. Masses are the same as they are on Earth, but weights are only a sixth of that on Earth.

Normal Forces

- When two objects come in contact with each other, each surface repels the other.
- As these forces are directed perpendicular to the surface, they are called "Normal Forces."
- The strength of a given normal force is dependent upon the circumstances. Its value can change as the circumstances change.

Force Diagrams

- To account for the various forces acting objects, we typically make a force diagram showing the forces acting on each object individually.



Example: A large ceramic planter when filled with dirt has a mass of 86.0 kg. A second identical dirt-filled planter is placed on top of it. A third dirt-filled planter, which has a mass of 16.5 kg, is placed on top. Determine A) the normal force of the floor on the bottom planter, and B) the normal force the second planter exerts on the dirt at the top of the bottom planter.

- Start by making a diagram of the problem.
- Next make a force diagram for each object in the problem.
    - Force diagrams must include every force that acts upon that object.
    - All objects will have weight in a gravitational field
    - At every point of contact between objects, each object experiences a normal force repelling it from the other. These are equal and opposite.

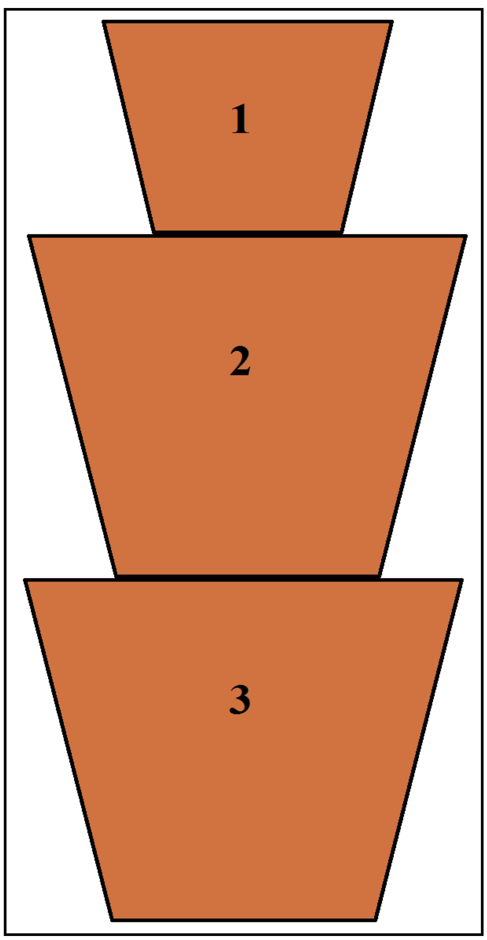
An object in contact with more than one object will experience more than one normal force.

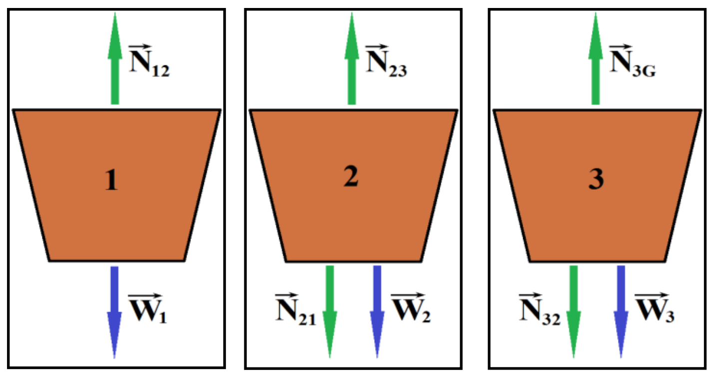
- At every point of contact between objects, each object experiences a friction force opposing its motion. These are also equal and opposite.

An object in contact with more than one object may experience more than one friction force.

- Any ropes, cords, chains, etc. will produce a force called tension (discussed later)
- Any other applied forces must also be included.

- In this problem, only vertical forces are relevant. The horizontal friction forces are zero as there are no other horizontal forces.





Starting with the top planter: Vertical forces must sum to zero.

$$N_{12} = W_1 = m_1 g = (16.5 \text{ kg})(9.80 \text{ m/s}^2) = 161.7 \text{ N}$$

Middle planter: Vertical forces must sum to zero.

The top planter creates a normal force pushing down.

$$N_{23} = N_{21} + W_2 = N_{12} + m_2 g = 161.7 \text{ N} + (86.0 \text{ kg})(9.80 \text{ m/s}^2) = 161.7 \text{ N} + 842.8 \text{ N} = 1004.5 \text{ N}$$

Bottom Planter: Vertical forces must sum to zero.

The middle planter creates a normal force pushing down.

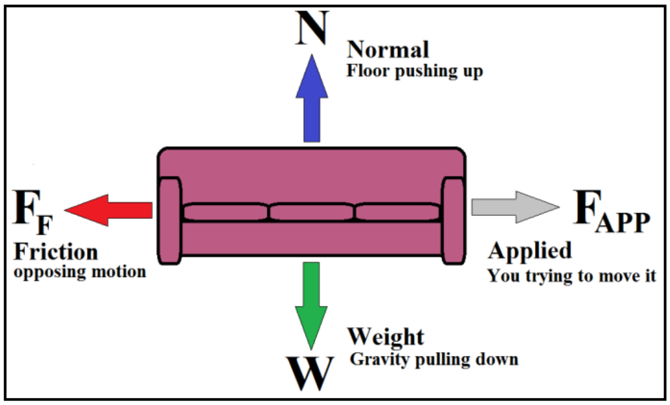
$$N_{3G} = N_{32} + W_3 = N_{23} + m_3 g = 1004.5 \text{ N} + (86.0 \text{ kg})(9.80 \text{ m/s}^2) = 1004.5 \text{ N} + 842.8 \text{ N} = 1847.3 \text{ N}$$

A) $N_{3G} = 1847.3$ N B) $N_{32} = N_{23} = 1004.5$ N

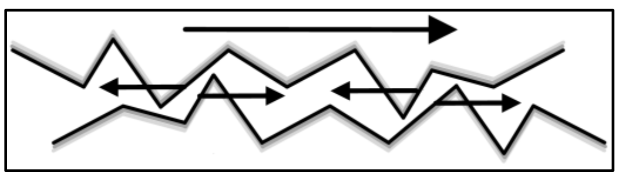
Friction

- When we push on an object (such as a couch) it won't move until a sufficient magnitude of force is applied.
- It doesn't move because the force of friction (static friction) matches the applied force and points in the opposite direction.

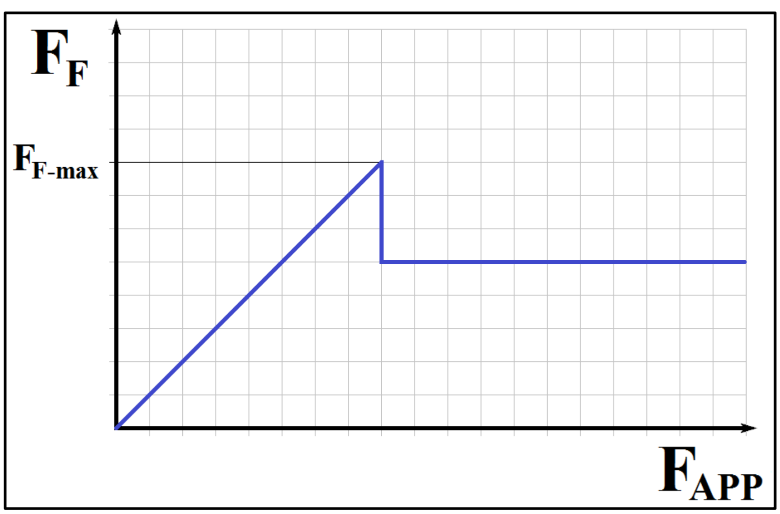
If the acceleration is zero (a =0), then the net force must be zero ($F_{NET} = 0$). For the net force to be equal to zero, the force of friction must be equal in magnitude to the applied force ($F_F = F_{APP}$) and opposite in direction.



- Friction occurs because on the small scale surfaces are rough and numerous tiny normal forces appear when you try to slide the surfaces across each other.



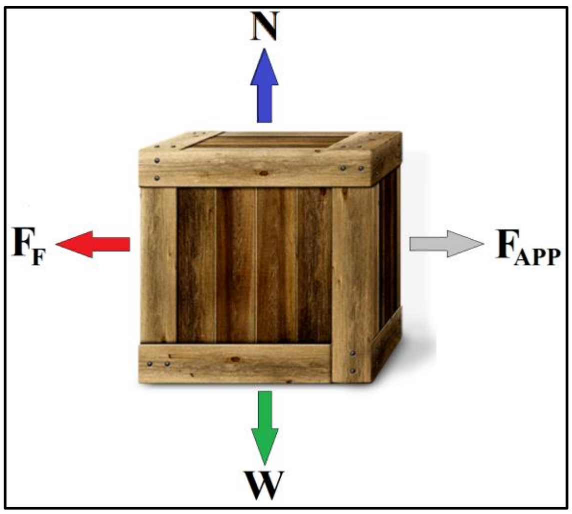
- Once sufficient force is applied to overcome static friction ($F_{F\text{-}Max}$), the force of friction takes on a constant value related to the normal force.



- Friction is parallel to the surface and always opposes motion.
- Solving problems with friction
    - If the object is known to be stationary, use $F_{NET} = 0$ to find F_F.

- If the object is known to be moving, use $F_F = \mu_k N$
  - N is the normal force related to that point of contact.
  - μ_k is the "coefficient of kinetic friction"
- If it is not known if the object is stationary or moving, find out.
  - Assume the object is stationary, and use $F_{NET} = 0$ to find F_F
  - Calculate $F_{F\text{-}Max} = \mu_s N$
    - N is the normal force related to that point of contact.
    - μ_s is the "coefficient of static friction"
  - If the calculated value of F_F exceeds $F_{F\text{-}Max}$ ($F_F > F_{F\text{-}Max}$), then the object is moving. The value of F_F previously calculated is not applicable. Set $F_F = \mu_k N$.
  - If the calculated value of F_F does not exceed $F_{F\text{-}Max}$ ($F_F \le F_{F\text{-}Max}$), then the object is stationary. Keep the value of F_F previously calculated (it is applicable).

Example: A block whose weight is 45.0N rests on a horizontal table. The coefficients of static and kinetic friction are 0.650 and 0.420 respectively. A horizontal force of 36.0N is applied to the block. Will the block move under influence of the force, and if so, what will be the blocks acceleration?



Is the crate moving? We don't know.
Assume it's stationary, find F_F, and compare it to $F_{F\text{-}Max}$.

For F_{NET} to be zero, $F_F = F_{APP} = 36.0$ N
$F_{F\text{-}Max} = \mu_S N = (0.650)(45.0\text{ N}) = 29.25$ N

Is the crate moving? Yes! 36.0 N > 29.25 N
We must use kinetic friction.

$F_F = \mu_k N = (0.420)(45.0\text{ N}) = 18.9$ N
$F_{NET\text{-}x} = F_{APP} - F_F = 36.0\text{ N} - 18.9\text{ N} = 17.1$ N

$m = W/g = (45.0\text{ N})/(9.80\text{ m/s}^2) = 4.5918$ kg
$a = a_x = F_{NET\text{-}x}/m = (17.1\text{ N})/(4.5918\text{ kg}) = 3.724\text{ m/s}^2$
$\Rightarrow 3.72\text{ m/s}^2$

Example: A block of mass M = 10.0 kg rests on an incline sloped at an angle θ = 30.0°. Friction's hold on the box is tentative, and the slightest touch will cause it to slide down the incline. Determine the force of friction, the normal force, and the coefficient of static friction.

First, 'sliding at the slightest touch' means $F_F = F_{F\text{-}max}$.
We will solve this problem three different ways.

Method 1: Brute force

Start by making a force diagram and breaking each vector into components.
Use three equations ($F_F = \mu_S N$, $\Sigma F_x = 0$, and $\Sigma F_y = 0$) to find three unknowns (F_F, μ_S, and N)

The angle of N is defined with respect to the y-axis.
You must use Sin(θ) or use geometry to find the angle with respect to the positive x-axis.

$\Sigma F_x = 0 \;\Rightarrow\; F_{Fx} = N_x \quad F_F\cdot Cos\,(\theta) = N\cdot Sin(\theta)$

$\mu_S\cdot N\cdot Cos\,(\theta) = N\cdot Sin(\theta)$

$\mu_S = N\cdot Sin(\theta)/N\cdot Cos\,(\theta)$

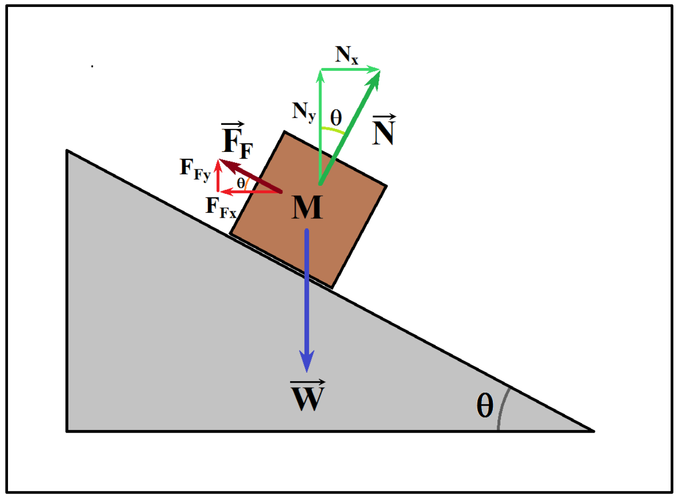
$\mu_S = Tan(\theta) = Tan\,(30.0°) = 0.57735 \;\Rightarrow\; 0.577$

$\Sigma F_y = 0 \;\Rightarrow\; F_{Fy} + N_y = W$

$F_F\cdot Sin(\theta) + N\cdot Cos\,(\theta) = Mg$

$\mu_S\cdot N\cdot Sin(\theta) + N\cdot Cos\,(\theta) = Mg$

$N\cdot[\mu_S\cdot Sin(\theta) + Cos\,(\theta)] = Mg$



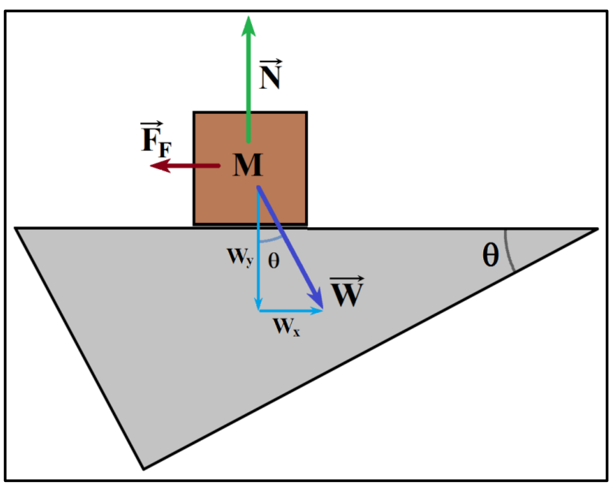
$N = Mg/[\mu_S\cdot Sin(\theta) + Cos\,(\theta)]$

$N = (10.0\ kg)(9.80\ m/s^2)[(0.57735)\cdot Sin(30.0°) + Cos\,(30.0°)] = 84.8705\ N \;\Rightarrow\; 84.9\ N$

$F_F = \mu_S\cdot N = (0.57735)(84.8705\ N) = 49.0\ N$

<u>Method 2</u>: Use a Different (Rotated) Reference Frame.

Start by making a force diagram and breaking each vector into components.
However, this time let the x-axis run parallel to the slope and the y-axis perpendicular.
This only leaves one vector (W) to break into components instead of two (N and F_F).
Use three equations ($F_F = \mu_S N$, $\Sigma F_x = 0$, and $\Sigma F_y = 0$) to find three unknowns (F_F, μ_S, and N)



$\Sigma F_x = 0 \;\Rightarrow\; F_F = W_x = mg\cdot Sin(\theta)$

The angle of W is defined with respect to the y-axis. You must use Sin(θ) or use geometry to find the angle with respect to the positive x-axis.

$F_F = (10.0\ kg)(9.80\ m/s^2)\cdot Sin(30.0°) = 49.0\ N$

$\Sigma F_y = 0 \;\Rightarrow\; N = W_y = mg\cdot Cos(\theta)$

The angle of W is defined with respect to the y-axis. You must use Cos(θ) or use geometry to find the angle with respect to the positive x-axis.

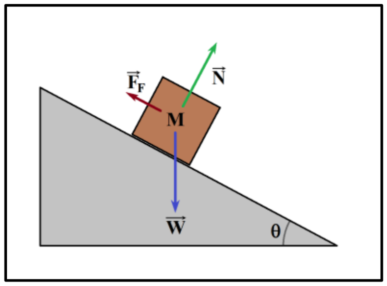
$N = (10.0\ kg)(9.80\ m/s^2)\cdot Cos(30.0°) = 84.8705\ N \;\Rightarrow\; 84.9\ N$

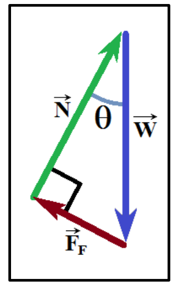
$F_F = \mu_S\cdot N \quad \mu_S = F_F/N = [mg\cdot Sin(\theta)] / [mg\cdot Cos(\theta)] = Tan(\theta) = Tan\,(30.0°) = 0.57735 \;\Rightarrow\; 0.577$

<u>Method 3</u>: Trigonometry

Start by making a force diagram. Upon seeing that 3 forces sum to zero, make a triangle out of them.

Upon finding N and F_F, use $F_F = \mu_S N$ to get μ_S.





$$N = W\cdot\text{Cos}(\theta) = mg\cdot\text{Cos}(\theta) = (10.0\text{ kg})(9.80\text{ m/s}^2)\cdot\text{Cos}(30.0°) = 84.8705\text{ N} \Rightarrow 84.9\text{ N}$$

$$F_F = W\cdot\text{Sin}(\theta) = mg\cdot\text{Sin}(\theta) = (10.0\text{ kg})(9.80\text{ m/s2})\cdot\text{Sin}(30.0°) = 49.0\text{ N}$$

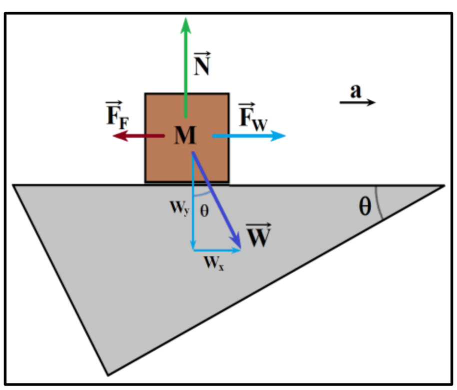
$$\mu_S = F_F/N = \text{Tan}(\theta) = \text{Tan}(30.0°) = 0.57735 \Rightarrow 0.577$$

On Solving Problems

- Solving the problem by making a triangle was the easiest solution, but this doesn't always work.
    - This can only be done when you can make a triangle.
    - It may not work in non-equilibrium or when a 4th force is present.
- Solving the problem by aligning the coordinate axes with the incline was easier.
    - It's only easier when more vectors align with these coordinate axes.
- Brute force is often the most difficult option, but it always works.
- Find the easiest way to solve each problem!

Example 21: A girl is skiing down a slope that is 30.0° with respect to the horizontal. A moderate wind is aiding the motion by providing a steady force of 105N that is parallel to the motion of the skier. The combined mass of the girl and skis is 65.0kg and the coefficient of kinetic friction between the skis and the snow is 0.150. How much time is required for the skier to travel down a 175m slope, starting from rest?

Start with a force diagram. Laying it flat leaves only one vector (W) to be broken into components. Use the force information to determine the acceleration and then do the kinematics.



This is a non-equilibrium situation $\Rightarrow \Sigma F_x = Ma \quad \Sigma F_y = 0$

$\Sigma F_y = 0 \Rightarrow N = W_y = W\cdot\text{Cos}(\theta) = Mg\cdot\text{Cos}(\theta)$

$F_F = \mu_k N = \mu_k Mg\cdot\text{Cos}(\theta)$

$\Sigma F_x = Ma \Rightarrow F_W + W_x - F_F = Ma$

$F_W + Mg\cdot\text{Sin}(\theta) - \mu_k Mg\cdot\text{Cos}(\theta) = Ma$

$a = F_W/M + g\cdot\text{Sin}(\theta) - \mu_k g\cdot\text{Cos}(\theta)$

$$a = (105\text{ N})/(65.0\text{ kg}) + (9.80\text{ m/s}^2)\cdot\text{Sin}(30°) - (0.150)(9.80\text{ m/s}^2)\cdot\text{Cos}(30°) = 5.24233\text{ m/s}^2$$

Extract kinematic data: Let $x_0 = 0$ $x = 175$ m $v_0 = 0$ $v =$ $a = 5.24233$ m/s^2 $t = ???$

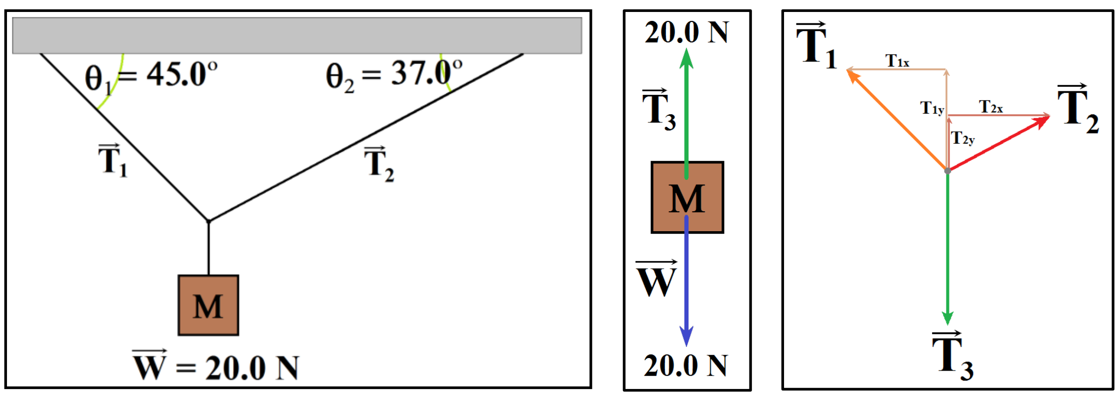
$$x = x_0 + v_0 t + ½\cdot at^2 = ½\cdot at^2 \qquad 2x = at^2 \qquad 2x/a = t^2$$

$$t = (2x/a)^{1/2} = [2(175m)/(5.24233)]^{1/2} = 8.17094\ s \Rightarrow 8.17\ s$$

Tension

- Tension is a force that is transmitted along the length of an object, often a flexible object such as a rope, string, or chain.
- Tension always "pulls" and never "pushes".
- The force is typically applied at the connections at each end, pulling along the length of the object.
- The force of tension is felt at each point in the interior of the object (pulled both ways)
- The direction this force is felt can be altered by pulleys. If the pulley has negligible mass and friction the tension is unaltered.

Example: A mass weighing 20.0N hangs from a system of strings as shown. Determine the tensions, T_1 and T_2.



Start by making a force diagram for the hanging mass, labeling the tension in the line above it 'T_3'.
Then make a force diagram at the point of intersection of the three tensions.
Break T_1 and T_2 into components.
This is static equilibrium so the forces must cancel. $\Sigma F_x = 0$ $\Sigma F_y = 0$

<u>Hanging Mass</u>: $\Sigma F_y = 0 \Rightarrow T_3 = W = 20.0\ N$

<u>Intersection (x-comp)</u>: $\Sigma F_x = 0 \Rightarrow T_{1x} = T_{2x}$ $\quad T_2 Cos(37.0°) = T_1 Cos(45.0°)$ $\quad T_2 = 0.885394 \cdot T_1$

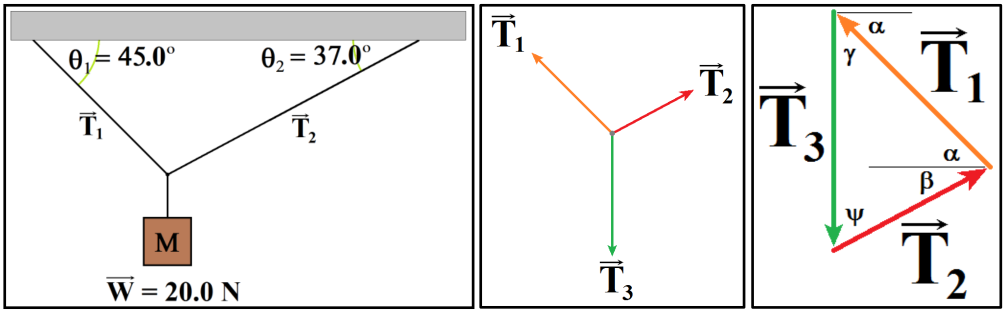
<u>Intersection (y-comp)</u>: $\Sigma F_y = 0 \Rightarrow T_3 = T_{1y} + T_{2y}$ $\quad T_3 = T_1 Sin(45.0°) + T_2 Sin(37.0°)$

$$T_3 = T_1 Sin(45.0°) + 0.885394 \cdot T_1 Sin(37.0°) = T_1[Sin(45.0°) + 0.885394 \cdot Sin(37.0°)]$$

$$20.0\ N = 1.23995 \cdot T_1 \qquad T_1 = 16.1297\ N \Rightarrow 16.1\ N$$

$$T_2 = 0.885394 \cdot T_1 = 0.885394 \cdot 16.1297\ N = 14.3\ N$$

Alternatively, we could have made a triangle and used the law of sines.



$\alpha = 45.0°$ (Given) $\beta = 37.0°$ (Given) $\gamma = 90° - \alpha = 45.0°$ $\psi = 90° - \beta = 53.0°$

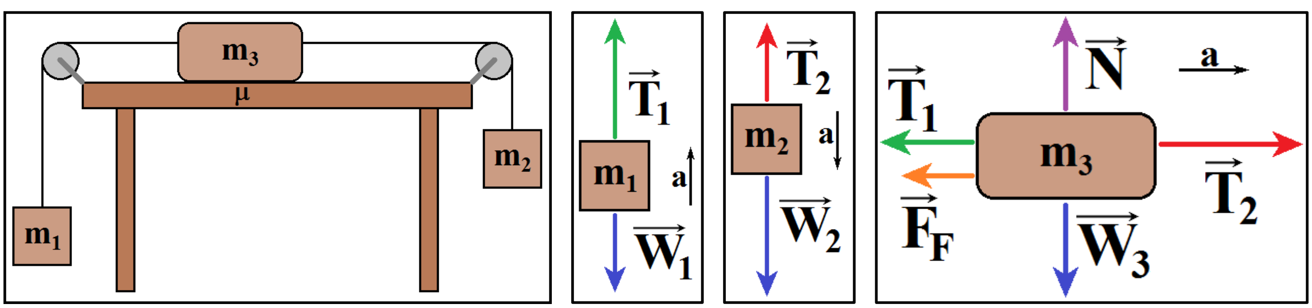
$$\frac{\text{Sin}\,\psi}{T_1} = \frac{\text{Sin}\,(\alpha+\beta)}{T_3} \qquad T_3 \cdot \text{Sin}\,\psi = T_1 \cdot \text{Sin}\,(\alpha+\beta)$$

$$T_1 = \frac{T_3 \cdot \text{Sin}\,\psi}{\text{Sin}\,(\alpha+\beta)} = \frac{(20.0\text{ N})\text{Sin}(53.0°)}{\text{Sin}(45.0°+37.0°)} = \frac{(20.0\text{ N})\text{Sin}(53.0°)}{\text{Sin}(82.0°)} = 16.1297\text{ N} \Rightarrow 16.1\text{ N}$$

$$\frac{\text{Sin}\,\gamma}{T_2} = \frac{\text{Sin}\,(\alpha+\beta)}{T_3} \qquad T_3 \cdot \text{Sin}\,\gamma = T_2 \cdot \text{Sin}\,(\alpha+\beta)$$

$$T_2 = \frac{T_3 \cdot \text{Sin}\,\gamma}{\text{Sin}\,(\alpha+\beta)} = \frac{(20.0\text{ N})\text{Sin}(45.0°)}{\text{Sin}(45.0°+37.0°)} = \frac{(20.0\text{ N})\text{Sin}(45.0°)}{\text{Sin}(82.0°)} = 14.2811\text{ N} \Rightarrow 14.3\text{ N}$$

Example: A box of mass $m_3 = 10.0$ kg sits on a table as shown. On one side a cord connects it to a hanging weight of mass $m_2 = 25.0$ kg. On the other side a cord attaches it to a hanging weight of mass $m_1 = 5.00$ kg. If the coefficient of kinetic friction between the box and table is 0.300, determine the acceleration of the system. Assume the mass and friction of the pulleys is negligible.



Can we go straight to m_3, treating the two tensions as m_1g and m_2g?

No! $a \neq 0 \Rightarrow T \neq W$.

What direction is m_3 accelerating? To the right ($m_2 > m_1$). So, friction points to the left.

Start by making force diagrams for all 3 masses.

There are 4 unknowns (a, F_F, T_1, and T_2).

We need 4 equations (ΣF_y for m_1, ΣF_y for m_2, ΣF_y for m_3, ΣF_x for m_3).

$$\Sigma F_y = m_1a: \quad T_1 - m_1g = m_1a \qquad T_1 = m_1a + m_1g$$

$$\Sigma F_y = m_2a: \quad m_2g - T_2 = m_2a \qquad T_2 = m_2g - m_2a$$

m_2 accelerates downward (typically negative), but a is being treated as positive.

You either need to make down positive or use '-a' as the acceleration to correct for this.

$$\Sigma F_y = 0 \text{ for } m_3: \quad N = W_3 = m_3 g \qquad F_F = \mu N = \mu m_3 g$$

$$\Sigma F_x = m_3 a: \quad T_2 - T_1 - F_F = m_3 a \qquad m_2 g - m_2 a - m_1 a - m_1 g - \mu m_3 g = m_3 a$$

$$m_2 g - m_1 g - \mu m_3 g = m_1 a + m_2 a + m_3 a$$

$$(m_2 - m_1 - \mu m_3)g = (m_1 + m_2 + m_3)a$$

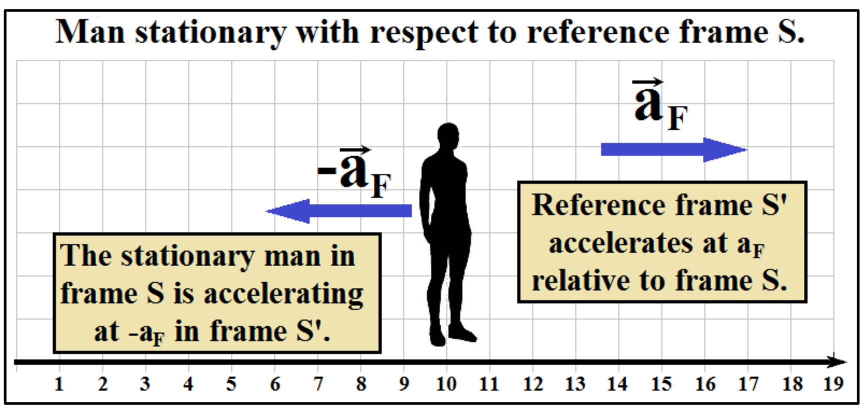
$$a = (m_2 - m_1 - \mu m_3)g / (m_1 + m_2 + m_3)$$

$$a = (25.0\text{ kg} - 5.00\text{ kg} - 0.300 \cdot 10.0\text{ kg}) \cdot (9.80\text{ m/s}^2) / (5.00\text{ kg} + 25.0\text{ kg} + 10.0\text{ kg}) = 4.17\text{ m/s}^2$$

Apparent Gravity

- One method of dealing with an accelerating reference frame (a non-inertial reference frame) is to 'package' the effects created by acceleration in combination with gravity (g) into apparent gravity (g_{app}).
- The acceleration (a_F) of a non-inertial reference frame relative to an inertial reference frame, causes every object in that inertial reference frame to have an additional acceleration ($-a_F$) when viewed in the non inertial frame.

For example, the man below is stationary with respect to his original reference frame. If our coordinate system accelerates to the right at a_F, then he must accelerate in the opposite direction at $-a_F$.



- This additional acceleration ($-a_F$) can be combined (vector addition) to gravity to get apparent gravity.

$$\vec{g}_{APP} = \vec{g} - \vec{a}_F$$

- Problems in the non-inertial frame can then be treated as if they were an inertial frame with a new value (and possibly direction) of gravity.

Example: A person is holding a 5.0 kg package in an elevator that begins accelerating upwards as 1.2 m/s^2. What force must the person exert to hold the package in the same relative position?

You could treat this as a person, elevator and package accelerating in an inertial frame.

$$F - mg = ma \Rightarrow F = ma + mg = (5.0\text{ kg})(1.2\text{ m/s}^2) + (5.0\text{ kg})(9.8\text{ m/s}^2) = 6N + 49N = 55\text{ N}$$

Or you could treat this as a person and package stationary in an accelerating reference frame.

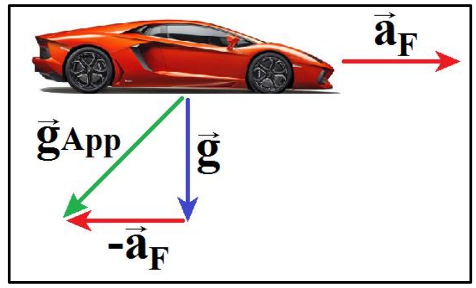
Note that as a_F points upwards, - a_F points downward (in the same direction as gravity). This indicates that you should add the magnitudes.

$$g_{app} = g - a_F = 9.8\ m/s^2 - (-1.2\ m/s^2) = 11\ m/s^2$$

$$W_{app} = mg_{app} = (5.0\ kg)\ (11\ m/s^2) = 55N$$

Conceptual Example: When you are in an accelerating car, you feel like you are being pushed back into your seat.

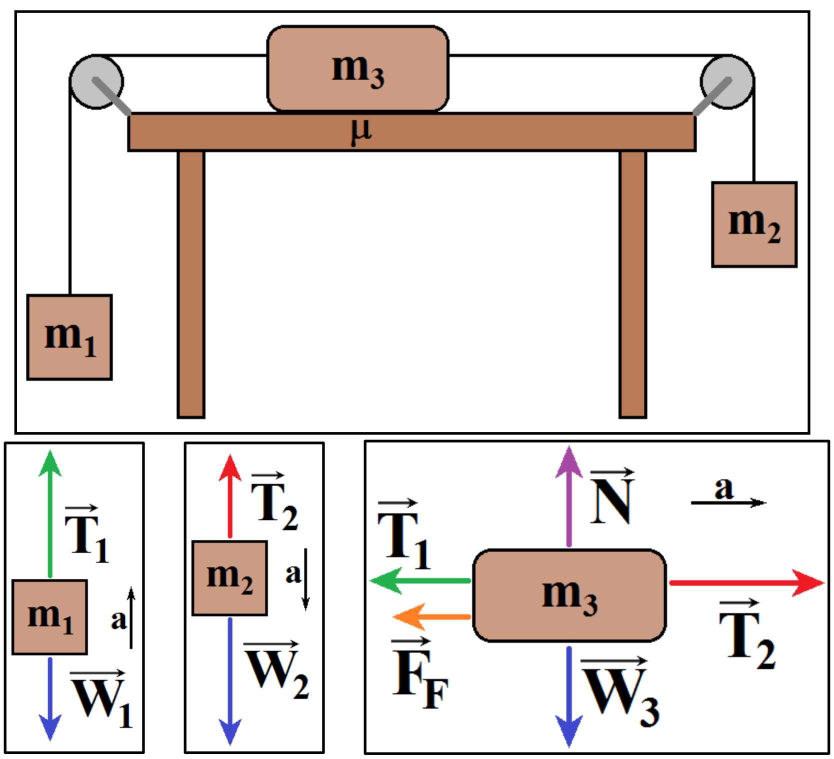
$$|g_{APP}| = \sqrt{g^2 + a_F^2}$$



Please don't measure the acceleration of your car by noting the angle of something hanging from the mirror while driving!

Example: A box of mass m_3 = 10.0 kg sits on a table as shown. On one side a cord connects it to a hanging weight of mass m_2 = 25.0 kg. On the other side a cord attaches it to a hanging weight of mass m_1 = 5.00 kg. The coefficient of kinetic friction between the box and table is 0.300. *If the table is placed in an elevator that is accelerating downward at 1.20 m/s²*, determine the horizontal acceleration of the box on the table. Assume the mass and friction of the pulleys is negligible.

This is identical to the problem we solved earlier with one exception. Now it's in an elevator accelerating downward.



We could solve this without apparent gravity, making modifications in each force diagram.

Decrease the acceleration of m_1 (in $\Sigma F_y = m_1 a$): $T_1 - m_1 g = m_1(a - 1.20\ m/s^2)$

Increase the acceleration of m_2 (in $\Sigma F_y = m_2 a$): $m_2 g - T_2 = m_2(a + 1.20\ m/s^2)$

Account for the acceleration of m_3 (ΣF_y is not zero): $W_3 - N = m_3(1.20\ m/s^2)$

Which changes friction to: $F_F = \mu N = \mu m_3 g - \mu m_3(1.20\ m/s^2)$

Then solve for a after plugging T_1, T_2, and F_F into: $T_2 - T_1 - F_F = m_3 a$

Or we can solve it as we did before and replace g with $g_{app} = g - a_F$

$$g_{app} = g - a_F = 9.80\ m/s^2 - 1.20\ m/s^2 = 8.6\ m/s^2$$

$$a = (m_2 - m_1 - \mu m_3)g_{app} / (m_1 + m_2 + m_3)$$

$$a = (25.0\ kg - 5.00\ kg - 0.300 \cdot 10.0\ kg) \cdot (8.60\ m/s^2) / (5.00\ kg + 25.0\ kg + 10.0\ kg) = 3.655\ m/s^2$$

FYI, one of the major breakthroughs of Albert Einstein was the 'equivalence principle', essentially stating that gravity was no different than any other acceleration. This lead to his prediction that light would bend due to gravity and eventually to general relativity.